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1972 J. Phys. A: Gen. Phys. 5 444

(http://iopscience.iop.org/0022-3689/5/3/013)

J. Phys. A: Gen. Phys., Vol. 5, March 1972. Printed in Great Britain

Underground range fluctuations of high energy muons for various energy loss parameters

E KIRALY†, P KIRALY† and J L OSBORNE
Department of Physics, University of Durham, Durham City, UK

MS received 12 August 1971

Abstract. The various methods by which the problem of range fluctuations of muons underground can be solved are discussed. Using an analytical solution due to Nishimura, curves are derived from which the enhancement factor due to fluctuations of the underground muon intensity can quickly be obtained, for a wide range of values of the muon energy loss parameters. A survey is given of previously published work in this field and discrepancies are resolved.

1. Introduction

Although there have been a number of studies of the problem of range fluctuations of high energy muons there is still considerable interest in the subject for several reasons.

Some cosmic ray results (eg the X process of the Utah group (Bergeson *et al* 1968) and the phenomenon of horizontal extensive air showers observed in Tokyo by Nagano *et al* (1970)) suggest that the muon exhibits anomalous behaviour at high energies. A new form of muon interaction, in addition to the normal electromagnetic interactions, would imply rather large and dramatic changes in some of the energy loss processes. Bergeson *et al* called for a photonuclear energy loss up to ten times larger than the conventional value to reconcile their proposal of a directly' produced muon component with the observed depth–intensity measurements. Although Kiraly and Wolfendale (1970) have shown that the uncertainties in the primary cosmic ray spectrum make it unnecessary to demand such a large increase, the question will remain open until the muon sea level spectrum beyond 1 TeV is directly measured by the large magnetic spectrometers now under construction. In order to obtain information on the energy loss from a comparison of this spectrum with the depth–intensity measurements, range fluctuation calculations for a wide range of parameter values are needed.

For the conventional interaction processes recent calculations have given somewhat different values for some of the energy loss parameters. For pair production losses the earlier workers used expressions due to Mando and Ronchi (1952) while Meyer *et al* (1970) and Bergamasco and Picchi (1971) have used the more recent values of Kelner and Kotov (1968). The latter values are 40 % greater than the former. Another problem is that there is still considerable uncertainty in the energy losses due to photonuclear interactions. Cassiday (1971) has resolved some of the discrepancies in the predictions of photonuclear energy loss but assumptions have to be made about the extrapolation of the measured photon–nucleon cross section above 20 GeV and the A dependence of

† On leave from Central Research Institute for Physics, Budapest.

the photon–nucleus cross section. The energy loss also depends upon the type of rock under which intensity measurements are made. The losses due to bremsstrahlung and pair production are proportional to z^2/A for the rock. The calculations to date have been made either for 'standard' rock with $z^2/A = 5{\cdot}5$ or Kolar rock $z^2/A = 6{\cdot}3$ (in the rock overburden in the Kolar gold fields, where many underground intensity measurements have been made). These are all reasons for wishing to know the sensitivity of the range fluctuation effects to the values of the energy loss parameters.

In the previously published work a number of treatments of the range fluctuation problem have given somewhat discordant results. It is difficult to tell whether this is due to the different values used for the energy loss parameters or the approximations inherent in the various methods of approach to the problem. In what follows, comparison with other workers is made in some detail and reasons advanced for the more outstanding differences.

The methods of calculation adopted by the various authors fall into three main groups. Hayman *et al* (1963), Osborne *et al* (1968) and Bergamasco and Picchi (1971) have used the Monte Carlo approach, Miyake *et al* (1964), Oda and Murayama (1965) and Meyer *et al* (1970) have made numerical calculations, while analytical solutions have been obtained by Zatsepin and Mikhalchi (1962, 1965), Nishimura (1964, 1965) and Kobayakawa (1967). The three methods have the following advantages and disadvantages. Both the numerical and Monte Carlo methods lead to $P(D, E)$, the arrival probability of a muon, with energy E at the surface, at depth D underground. From this the intensity at depth D underground, with fluctuations taken into account, is given by

$$I_{\mathrm{F}}(D) = \int_{E_{\min}}^{\infty} P(D, E)N(E)\,\mathrm{d}E \tag{1}$$

where $E_{\min}$ is the minimum possible surface energy for muons to reach depth D and $N(E)\,\mathrm{d}E$ is the differential energy spectrum of muons at the surface and can be of any form. The mean rate of energy loss of muons can be written as

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = a + b_{\mathrm{t}}E \tag{2}$$

where the first term representing ionization loss has a component rising logarithmically with energy and $b_{\mathrm{t}}E$ represents the total losses due to bremsstrahlung, pair production and photonuclear interaction. According to the conventional theory of muon interactions the factor b_{t} also changes slowly with energy reaching an asymptotic value in the region of 1 to 10 TeV. The Monte Carlo approach allows one to take into account the variation of b_{t} and a with energy and to use energy dependent differential cross sections. The disadvantage lies in the large number of particles that need to be followed to achieve good statistical accuracy, since small values of the arrival probability $P(D, E)$ contribute strongly to the intensity in (1) due to the sharply falling muon spectrum. The numerical method also allows one to take energy dependent values of b_{t} and a and the differential cross sections. Although it is free from statistical uncertainties the width of the depth intervals has to be small if the effect of fluctuations is not to be underestimated and this leads to extensive computations at great depths.

The analytical solution requires several approximations to be made. The energy spectrum of muons at the surface is represented by a power law with constant exponent and the energy loss parameters a and b_{t} are taken as constants. The differential probability of a muon transferring a fraction v of its energy in an interaction $\phi(v)\,\mathrm{d}v$ is assumed to be independent of energy also. In the solution given by Zatsepin and Mikhalchi (1962) it is further assumed that for fluctuating losses $\phi(v)\,dv = dv/v$. The formulation of Nishimura (1964) on the other hand allows more complicated expression for $\phi(v)\,dv$ but is restricted to integral values of the exponent of the muon spectrum. Kobayakawa (1967) has shown how the method of Nishimura can be modified to take into account a specific energy dependence by using, for parameters, effective constant values that change with energy and depth underground.

All of the calculations so far, except those of Zatsepin and Mikhalchi have been made for given values of the energy loss parameters, chosen by the authors as the current best estimates for the particular rock under consideration (Hayman *et al* (1963) used two sets of values corresponding to upper and lower limits estimated by them). As mentioned earlier, it is important to know, however, how the muon depth intensity curve will change as the energy loss parameters vary over a wide range of values. The Monte Carlo and numerical methods, although giving the most accurate and immediately applicable results, are too cumbersome to apply in this case. Therefore in what follows we use the analytical method of Nishimura.

2. Calculation of the enhancement factor due to fluctuations

If the integral spectrum of muons at the surface is

$$N(>E) = KE^{-\gamma} \tag{3}$$

then, using the mean energy loss relation (2) and assuming a and b_t constant, the intensity of muons at depth D ignoring the effect of fluctuations is given by

$$I_{NF}(D) = K\left(\frac{a}{b_t}(\exp(b_t D)-1)\right)^{-\gamma}. \tag{4}$$

Nishimura has shown that when fluctuations are taken into account the intensity is given by

$$I_F(D) = \frac{K}{\gamma!}a^{-\gamma}\sum_{n=0}^{\infty}\frac{\partial A(\gamma+n)}{\partial\gamma}\exp\{-A(\gamma+n)D\}\prod_{i=1}^{\gamma-1}(A(\gamma+n)-A(i)) \tag{5}$$

for integral values of γ, where

$$A(s) = \int_0^1 \{1-(1-v)^s\}\,[b_b\phi_b(v)+b_p\phi_p(v)+b_n\phi_n(v)]\,dv. \tag{6}$$

The term in square brackets is the probability per unit depth of a muon transferring a fraction v of its energy by bremsstrahlung, pair production or photonuclear interaction. Equation (6) may be rewritten as

$$A(s) = b_t a_t(s) = b_t(b'_b a_b(s)+b'_p a_p(s)+b'_n a_n(s)) \tag{7}$$

where $b'_b = b_b/b_t$ is the fractional contribution of bremsstrahlung to the part of the energy loss which is proportional to energy and

$$a_b(s) = \int_0^1 \{1-(1-v)^s\}\phi_b(v)\,dv \qquad \text{etc.} \tag{8}$$

It is useful now to define an enhancement factor which is the ratio of the intensity obtained with fluctuations to that without. Thus

$$F(D, \gamma) = \frac{I_{\mathrm{F}}(D)}{I_{\mathrm{NF}}(D)}. \tag{9}$$

If we now substitute (4), (5) and (7) in (9) at the same time expressing the depth in units of $1/b_{\mathrm{t}}$ (ie $d = b_{\mathrm{t}}D$)† the enhancement factor becomes

$$F(d, \gamma) = \frac{1}{\gamma!}(\exp(d)-1)^{\gamma} \sum_{n=0}^{\infty} \frac{\partial a_{\mathrm{t}}(\gamma+n)}{\partial \gamma} \exp\{-a_{\mathrm{t}}(\gamma+n)d\} \times \prod_{i=1}^{\gamma-1} (a_{\mathrm{t}}(\gamma+n)-a_{\mathrm{t}}(i)). \tag{10}$$

From this expression we see that the enhancement factor is independent of the ionization loss coefficient a and, provided that depth is written in units of $1/b_{\mathrm{t}}$, it is also independent of the magnitude of b_{t}. The approximate nature of the analytical solution is due to the necessary assumption that b_{b}, $\phi_{\mathrm{b}}(v)$ etc are independent of energy. This is sufficiently accurate in the energy region above 1 TeV where the fluctuation effects become important. The values of $a_{\mathrm{b}}(s)$, $a_{\mathrm{p}}(s)$ and $a_{\mathrm{n}}(s)$ are calculated as follows.

For bremsstrahlung we take

$$a_{\mathrm{b}}(s) = \int_0^1 \{1-(1-v)^s\}\frac{1}{v}\left(\frac{4}{3}+v^2-\frac{4}{3}v\right) \mathrm{d}v \tag{11}$$

corresponding to the complete screening approximation.

For photonuclear interactions

$$a_{\mathrm{n}}(s) = \int_0^1 \{1-(1-v)^s\}\frac{1}{v}\ln\frac{1}{v}\,\mathrm{d}v. \tag{12}$$

The expression for $\phi_{\mathrm{n}}(v)$ used here is derived from the Williams–Weizsacker relation for the virtual photon flux.

For pair production $\phi_{\mathrm{p}}(v) \propto 1/v^3$ so that the probability of appreciable energy losses in a single interaction is small. If we set $\phi_{\mathrm{p}}(v) = \delta(v_0-v)/v_0$ and then let v_0 tend to zero we obtain $a_{\mathrm{p}}(s) = s$ corresponding to a nonfluctuating energy loss due to pair production. We use this as a sufficiently accurate approximation. (Setting $A(s) = b_{\mathrm{t}}s$ in (5) reduces it to the nonfluctuating expression (4).)

Values of $a_{\mathrm{b}}(s)$, $a_{\mathrm{p}}(s)$ and $a_{\mathrm{n}}(s)$ are given in figure 1 for s running from 1 to 30. For $d = 1$ the summation in (10) converges sufficiently rapidly that higher terms than the thirtieth can be neglected. The convergence improves as d increases and for $d = 4$ only 10 terms are needed. Because of the smooth variation of $a_{\mathrm{t}}(s)$ with s one can write the term

$$\frac{\partial}{\partial \gamma}a_{\mathrm{t}}(\gamma+n) \simeq \frac{a_{\mathrm{t}}(\gamma+n+1)-a_{\mathrm{t}}(\gamma+n-1)}{2}. \tag{13}$$

We also show in figure 1 the values of $a(s)$ obtained for $\phi(v) \propto 1/v$. This approximation was used for the bremsstrahlung losses by Zatsepin and Mikhalchi (1962) and Hayman *et al* (1963). They fall below the values $a_{\mathrm{b}}(s)$ and this will give an overestimate of the effect of fluctuations.

† Nishimura expresses the depth in cascade units (ie $1/b_{\mathrm{b}}$).

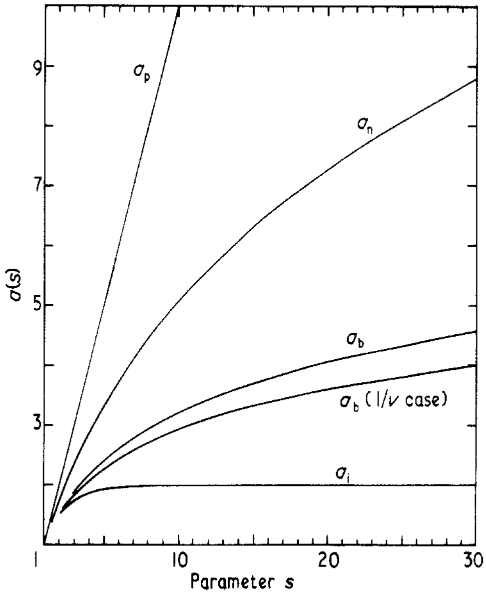

Figure 1. Parameters used in calculating the enhancement factor due to fluctuations in muon range. a_p pair production losses treated as a nonfluctuating process; a_n photonuclear interaction losses; a_b bremsstrahlung losses; a_b($1/v$ case) bremsstrahlung losses under the approximation $\phi(v) \propto 1/v$; a_i losses due to interactions with a constant inelasticity of 0·5.

It is interesting to consider muons interacting with constant inelasticity v_0. In this case there would be an additional term in equation (7) involving $a_i(s) = \{1-(1-v_0)^s\}/v_0$. The curve for inelasticity equal to 0·5 is given in figure 1. For a completely inelastic interaction where the muon loses all of its energy $a_i = 1$ for all s. Adding a constant value A_i to $A(s)$ in (5) is equivalent to multiplying $I_F(D)$ by $\exp(-A_iD)$ where A_i is the probability per unit depth of a completely inelastic interaction.

One may compare the values of $a(s)$ in figure 1 with the equivalent values tabulated up to $s = 19$ by Kobayakawa (1967). His value for bremsstrahlung with complete screening is identical to ours. Kobayakawa's accurate expression for $\phi_p(v)$ leads to a more slowly increasing $a_p(s)$. For example, at $s = 19$ the value is $a_p = 16{\cdot}4$. He finds that pair production gives about 7 % of the total fluctuation effect while we have ignored fluctuations in this case. He uses a much more complicated expression for $\phi_n(v)$ which results in $a_n(s)$ which is weakly energy dependent. In the region 1 to 10 TeV his $a_n(19)$ is approximately 6·5 while our value is 7·09. We will, therefore, obtain a slightly smaller enhancement due to fluctuations in the photonuclear energy loss than Kobayakawa.

3. Dependence of the enhancement on the parameters of energy loss

We have shown that the enhancement factor $F(d, \gamma)$ depends only on the relative values b'_b, b'_p and b'_n.

In figures 2, 3 and 4 this dependence is illustrated. The upper curves show $F(d, \gamma) - 1$ for $b'_n = 0$ with b'_p/b'_b varying from 0·75 to 1·50. The pair production energy loss of

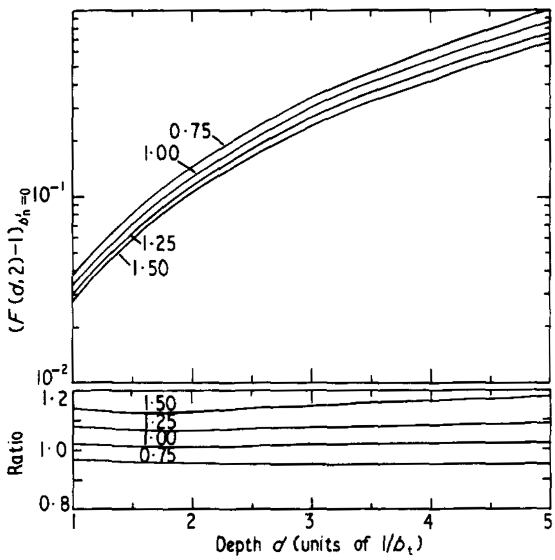

Figure 2. Enhancement factor due to fluctuations in muon range for $\gamma = 2$. Upper curves, enhancement factors for $b'_n = 0$ and various values of b'_p/b'_b (shown on the figure). Lower curves, ratios by which the corresponding upper curves must be multiplied to obtain $F(d, \gamma)-1$ for $b'_n = 0{\cdot}5$.

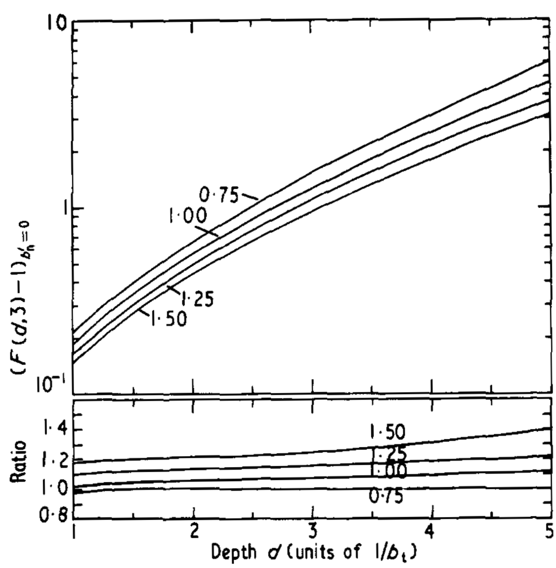

Figure 3. Enhancement factor due to fluctuations in muon range for $\gamma = 3$ (upper and lower curves as for figure 2).

Mando and Ronchi leads to a value of about 0·9 for the ratio while the Kelner and Kotov evaluation leads to approximately 1·3.

The lower curves show the ratio of $F(d, \gamma)-1$ for $b'_n = 0{\cdot}5$ to $F(d, \gamma)-1$ for $b'_n = 0$ as a function of the ratio b'_p/b'_b. The reason that the effect of changing b'_n is so small is that $a_n(s)$ is close to the mean of $a_b(s)$ and $a_p(s)$ so that $a_t(s)$ is insensitive to b'_n. 0·5 is a

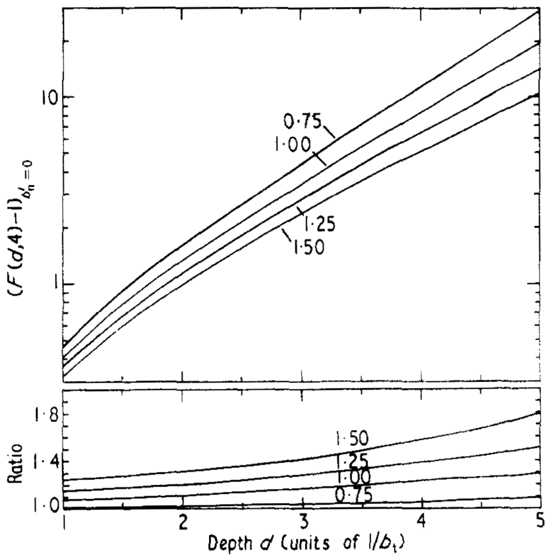

Figure 4. Enhancement factor due to fluctuations in muon range for $\gamma = 4$ (upper and lower curves as for figure 2).

generous upper limit to b_n'. To find the value of $F(d, \gamma)$ for given b_p', b_b' and b_n' first obtain the value for $b_n' = 0$ from the upper curves and then correct it for a finite b_n' using the lower curves. A linear interpolation can be used for values of b_n' less than 0·5.

The enhancement factors as they stand can only be used for incident power law spectra with constant integral exponents. A plot of $\lg(F(d, \gamma))$ against γ allows non-integral values of γ to be interpolated simply. For an incident spectrum with varying exponent the procedure is as follows. The spectrum is represented as a linear combination of power law functions

$$N(>E) = \sum_i K_i E^{-\gamma_i} \tag{14}$$

where the coefficients K_i need not be all positive. Then

$$I_F(D) = \sum_i K_i \left(\frac{a}{b_t} (\exp(b_t D) - 1) \right)^{-\gamma_i} F(D, \gamma_i). \tag{15}$$

The representation (14) need only be precise over a limited range of energy. This can be seen from equation (1) where the integrand is significant over about one decade of energy only, due to the sharp decrease of $N(E)$ towards higher energies and of $P(D, E)$ towards low energies. Outside the important region the linear combination may strongly deviate from $N(E)$ but at the larger energy side it should decrease steeply enough so that one gets a very small contribution in (1).

4. Comparison with other workers

We have compared the enhancement factors given by previous workers with our $F(d, \gamma)$ in each case using the appropriate values of the energy loss parameters. We consider the agreement to be satisfactory if the two enhancement factors differ by less than 10% (this exceeds the statistical accuracy of measured muon intensities in the relevant depth region). We find satisfactory agreement with Hayman *et al* (1963), Oda and Murayama (1965) and Osborne *et al* (1968). The first gives slightly higher values but this is accounted for by their assumption that $\phi(v) \simeq 1/v$ which overestimates the effect of fluctuations. The enhancement factor of Osborne *et al* is rather lower at the largest depths. This is probably due to their taking too large a depth cell (100 hg cm^{-2}) in their Monte Carlo calculation. Re-running some of the calculations with a depth cell of 50 hg cm^{-2} results in a slightly bigger enhancement factor. For comparison with Kobayakawa (1967) we estimate the effective constant values of the energy loss parameters to be $b_b = 1{\cdot}67 \times 10^{-6}$, $b_p = 1{\cdot}53 \times 10^{-6}$ and $b_n = 0{\cdot}3 \times 10^{-6}$ g^{-1} cm^2. His $F(d, \gamma)$ increases rather faster with depth than ours but agreement is satisfactory down to 10 000 hg cm^{-2}. The enhancement factors of Miyake *et al* (1964) are significantly lower than ours for $\gamma = 2$ and 3. This is probably accounted for by their use of too large units of depth in their numerical calculations.

Zatsepin and Mickhalchi (1962, 1965) have given an analytical solution of the fluctuation problem. From their expressions and tabulated values one can obtain the enhancement factor as a function of γ and the ratio b_p/b_b (photonuclear interactions are ignored). We have compared our results with theirs for the case $b_p/b_b = 1$ and find satisfactory agreement over the whole range of depths when one takes into account that they have used the approximation $\phi_b(v) \propto 1/v$.

Menon and Ramana Murthy (1967) in their review article give values of the enhancement factor that they calculated from Zatsepin and Mikhalchi's formulae for $b_p/b_b = 0{\cdot}89$. These values are higher by up to a factor of two than the values we obtain from the same formulae and we believe them to be in error.

Nishimura (1964) gives values for the enhancement factor for $b_p = 2{\cdot}2 \times 10^{-6}$ and $b_b = 1{\cdot}7 \times 10^{-6}$ g^{-1} cm^2 and ignoring photonuclear losses. His values are systematically higher than ours for the same parameters and in fact agree closely with those that we obtain for $b_p/b_b = 0{\cdot}65$. He does not state explicitly what forms of the function $\phi(v)$ were used in evaluating $A(s)$. It seems probable that the fluctuations in pair production losses have been overestimated.

In the present work it is assumed that the ionization loss coefficient a is a constant. Nishimura (1965) has calculated the effect on the enhancement factor of using the true logarithmically increasing ionization loss $(2{\cdot}55 + 0{\cdot}076 \ln (E/1150\ \text{GeV}))$ MeV g^{-1} cm^2 as opposed to a constant value of 2·55 MeV g^{-1} cm^2. He concludes that the effect is significant, for instance, for $\gamma = 3$ and $d = 2$ the enhancement factor is increased by 18 % due to the logarithmic increase in ionization loss. It is difficult to see physically why the effect should be so large particularly in view of the fact that for a constant rate of ionization loss a, $F(\gamma, d)$ is independent of the value of a. To check this we have calculated the enhancement factor by the Monte Carlo method with and without the logarithmic increase in a, for $\gamma = 3$ and d ranging from 1·5 to 4 and do not substantiate this effect. We conclude that it is sufficiently accurate to compute the enhancement factor without regard to the variation of a. Although in calculating the intensity without fluctuations $I_{NF}(D)$, to which the enhancement factor is applied, one should use the accurate value of the ionization loss.

5. Conclusion

The analytical method in general gives satisfactory agreement with the enhancement factors obtained by previous workers when the same values of the energy loss parameters are used. In those cases where a discrepancy exists reasons for it have been advanced. We conclude that the approximations inherent in the analytical approach do not have a significant effect.

We have presented the results in such a way that the enhancement factors can easily be obtained for a wide range of values of the energy loss parameters. However, if the muon has no anomalous interaction, there is a restricted range of possible values for these parameters and from this we can obtain a probable uncertainty in the enhancement factors. For standard rock we take $b_b = 1{\cdot}77 \times 10^{-6}\ g^{-1}\ cm^2$. Values for b_p range from $1{\cdot}58 \times 10^{-6}\ g^{-1}\ cm^2$ from Mando and Ronchi to $2{\cdot}3 \times 10^{-6}\ g^{-1}\ cm^2$ from Kelner and Kotov. Cassiday gives the most likely value of b_n as $0{\cdot}21 \times 10^{-6}\ g^{-1}\ cm^2$ which is also the smallest of those that he quotes. It seems to us that b_n could be as large as $0{\cdot}7 \times 10^{-6}\ g^{-1}\ cm^2$ without involving any new form of interaction. Thus we have a minimum value $b_t = 3{\cdot}56 \times 10^{-6}\ g^{-1}\ cm^2$ and a maximum value $b_t = 4{\cdot}77 \times 10^{-6}\ g^{-1}\ cm^2$. The range of values of the enhancement factor corresponding to those is shown in figure 5.

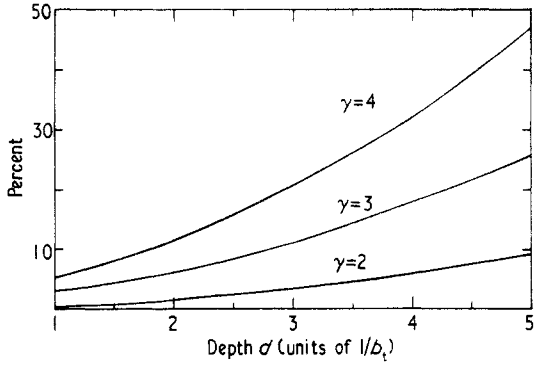

Figure 5. Percentage uncertainty in the enhancement factors due to our estimate of the possible range of values of the total muon energy loss $(F(d, \gamma)_{\min b_t} - F(d, \gamma)_{\max b_t})/F(d, \gamma)_{\max b_t}$.

It is important to realize that this does not reflect the full amount of the uncertainty in converting from the muon energy spectrum at the surface to the expected depth intensity curve. The intensity $I_{NF}(D)$ to which the enhancement factor is applied is very sensitive to the numerical value of b_t.

Acknowledgments

Miss M C Allcock is thanked for her help in running the computer programs.

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